The Asymptotic solution by the Heat Equation in Tube Domain

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Abstract:

In this paper; we study the heat equation in tube domain by using asymptotic consist of the regular function , bounding functions and corner functions , also solving this asymptotic through boundary and initial conditions .

1. Introduction

One of the mathematical branch which care by studying phenomena's which comes from the environment we live in .Several mathematical models are formed for many researchers to solve these phenomena's and really most of these models have a high ability to study (see Kreysing E. [5] and Smith D.R. [8]) .There are many authors studied the applied mathematics as physical mathematics . Levenshtam [4] studied the ordinary differential equations of the first order and the second degree . Technoff and Samarcki [7] studied only the partial differential equations of the first order and the second degree with boundary conditions . Eckhaus W. [4] studied the asymptotic of singular perturbations with initial conditions . Butozof B.F, Vaciliva A. B, Fideruak M.B. [3] studied the asymptotic method in the theorems of ordinary differential equation of the first order.

In this paper, we study the heat equation in tube domain ,and solving asymptotic consist of the regular function , boundary and corner functions by using boundary and initial conditions .

2. Main problem

In this paper, we study the following problem:

$$\varepsilon^2\frac{\partial u}{\partial t}-a(x)\left(\varepsilon^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y}\right)=\varepsilon^2 f(u,x,t)$$

$$(x,y,t)\in\Omega=(0<x<1)\,x\,(0<y<1)\,x\,(0<t\le T), \qquad (1)$$

$$u|_{t=0}=\varphi(x,y,\varepsilon),\ \ u|_{x=0}=\psi_1(y,t,\varepsilon),\ \ u|_{x=1}=\psi_2(y,t,\varepsilon), \qquad (2)$$

$$\frac{\partial u}{\partial y}-A\varepsilon^2 u|_{y=0}=0\ , \qquad \frac{\partial u}{\partial y}+A\varepsilon^2 u|_{y=1}=0. \qquad (3)$$

All functions in this problem consider sufficiently smoothness a (x), A > 0, and the condition (3) - agreement up to continuity, that is

$$\varphi(0,y,\varepsilon)=\psi_1(y,0,\varepsilon) \quad , \quad \varphi(1,y,\varepsilon)=\psi_2(y,0,\varepsilon).$$

3. Construction asympotic solution :

The solution expansion asymptotic problems (1) – (3), we will be constructing in the form of:

$$u(x,y,t,\varepsilon)=\sum_{k=0}^{\infty}\varepsilon^k\left(\bar{u}_k(x,y,t)+R_k(x,y,\tau)+Q_k(\xi,y,t)+Q_k^*(\xi^*,y,t)+P_k(\xi,y,\tau)+P_k^*(\xi^*,y,\tau)\right), \quad (4)$$

c) For each $x \in X$ and each open subset V of Y such that $x \in F^{-}(V)$, there is a regular open set U containing x such that $U \subset F^{-}(V)$.

d) For each $x \in X$ and each open subset V of Y such that $x \in F^{-}(V)$, there is a δ- open set U containing x such that $U \subset F^{-}(V)$.

4 Characterization of nearly compact spaces

In this section we characterized nearly compact spaces by nets with a well-ordered index set as domain . We will need the following lemma :

Lemma 4.1 :[4]

The following conditions are equivalent for a space X :

a) X is compact .

b) Each nest of non-empty closed subsets of X has a non-empty intersection .

c) Each net on X with a linearly ordered index set as domain has an accumulation point in X.

Theorem 4.2

A space X is nearly compact iff every net on X with a well-ordered index set as domain has a δ- accumulation point in X .

Proof :

$\Rightarrow$) follows from Lemma 3.4 .

$\Leftarrow$) Let $\{x_\beta : \beta \in D\}$ be a net on X with a linearly ordered index set D .

By Theorem (36) of Roitman [7] , D has a well-ordered cofinal subset E . Thus $\{x_\gamma : \gamma \in E\}$ is a subnet of $\{x_\beta : \beta \in D\}$ with a well-ordered index set .

From the hypothesis , $\{x_\gamma : \gamma \in E\}$ has a δ- accumulation point x_0 in X .Clearly x_0 is also a δ- accumulation point of the net $\{x_\beta : \beta \in D\}$.

So , by Lemma 4.1 , X_s is compact which means that X is nearly compact .

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